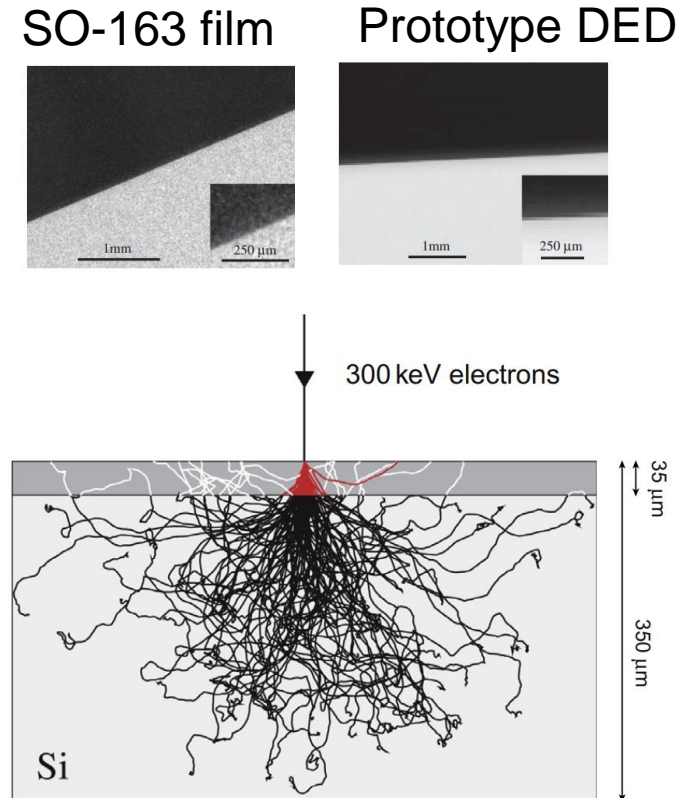


# Study Meeting 4: Introduction to the CTF

Zuben P. Brown & Prikshat Dadhwal

# Direct Electron Detector

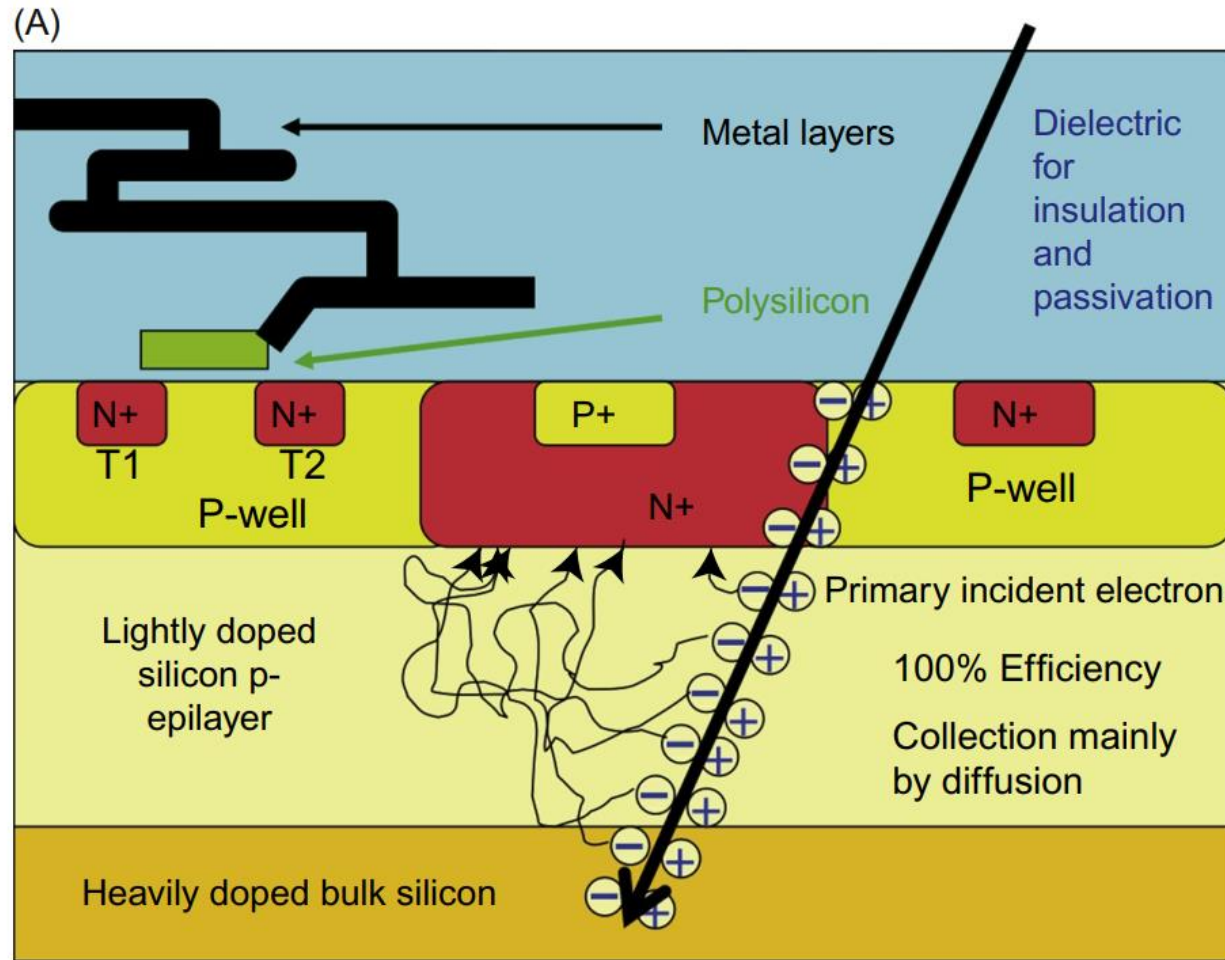
- DED are better
  - Fog noise (dark gain reference)
  - Back thinning
  - Motion correction
  - Reduced coincidence loss
- MTF: the camera envelope function
- DQE: Input SNR to output SNR



# Unanswered question

- Last time: Francisco asked about the effect of acceleration voltage on DED imaging:

# Direct Electron Detector



# Effect of electron voltage on DED

Interacts less with the sample

- Inelastic scattering lowered
- Reduced contrast

- Effect on back scattering?

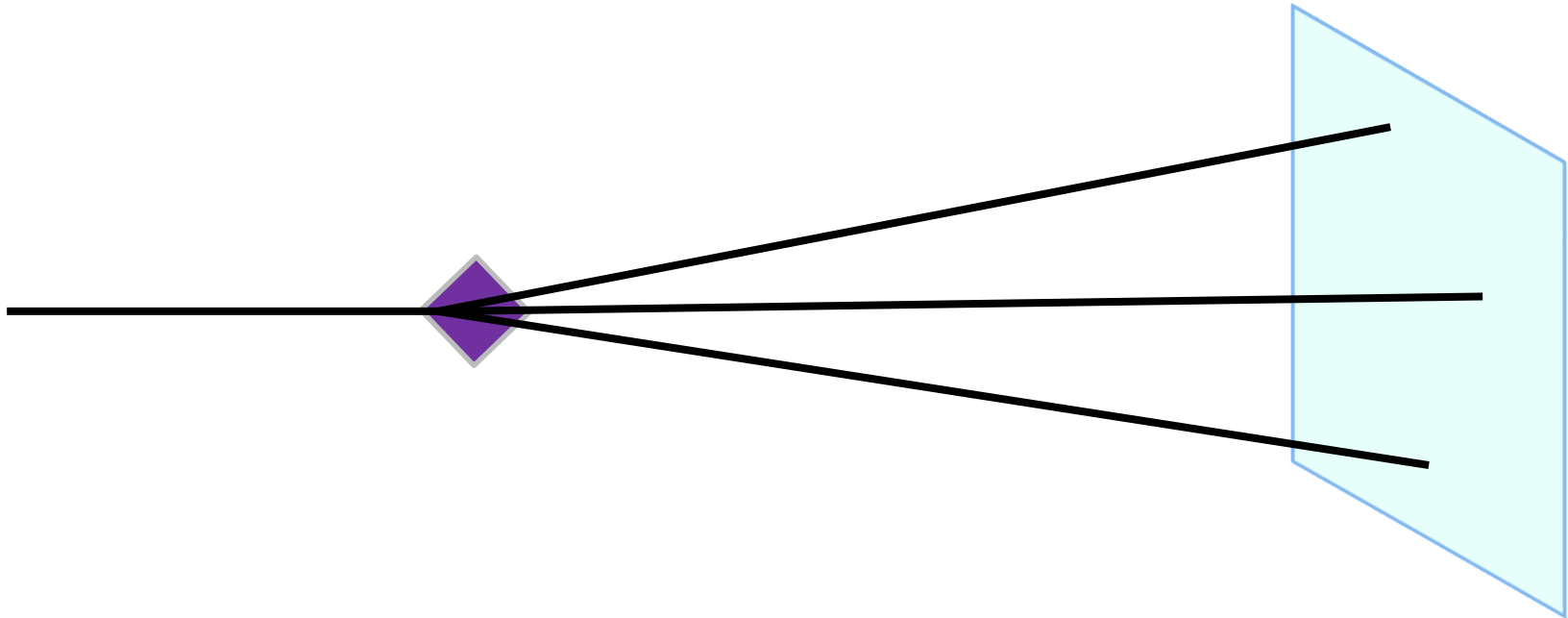
- Interacts less with the camera

- Shorter time to interact with camera
- Higher chance of coincidence loss (no, may have voltage difference below the cut off inherent to the camera)

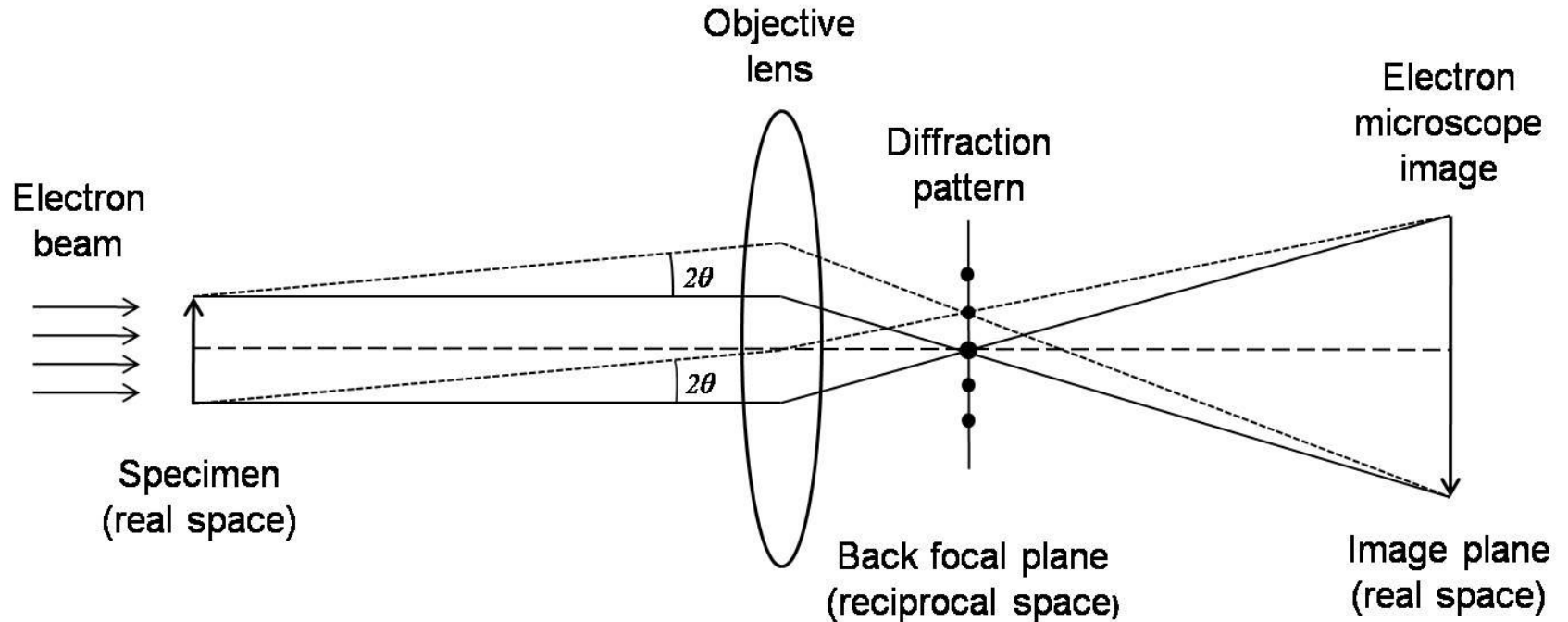
# Goals for today

- Image formation in EM
- The contrast transfer theory
- CTF equation
- Effect of various parameters on the CTF
- Why CTF estimation matters
- Envelope functions

# Diffraction

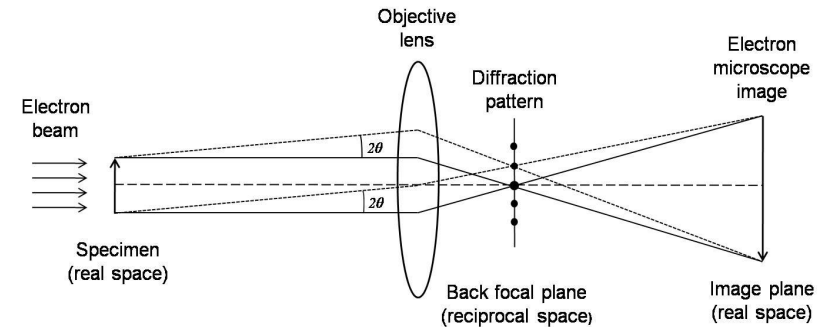
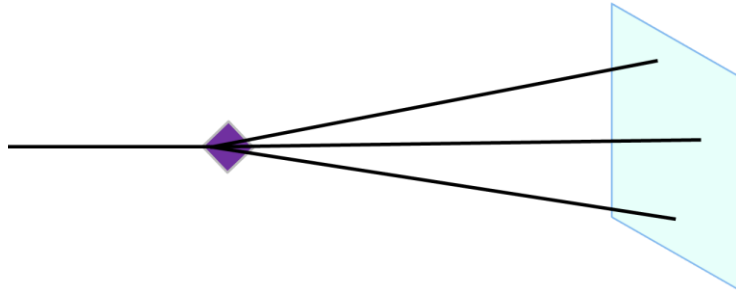


# Image formation in EM





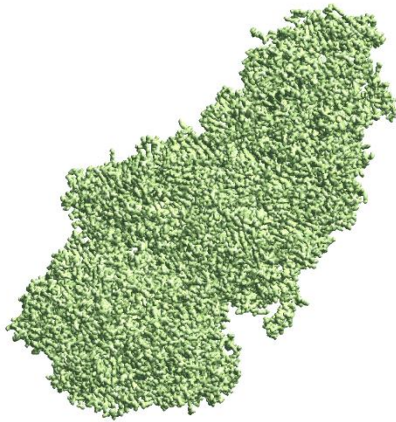
# X-ray vs. EM image formation



- Amplitude contrast
- Phase contrast
- EM uses lenses so we also get phase information
  - Interaction between the scattered and unscattered beam

# Projection of 3D density

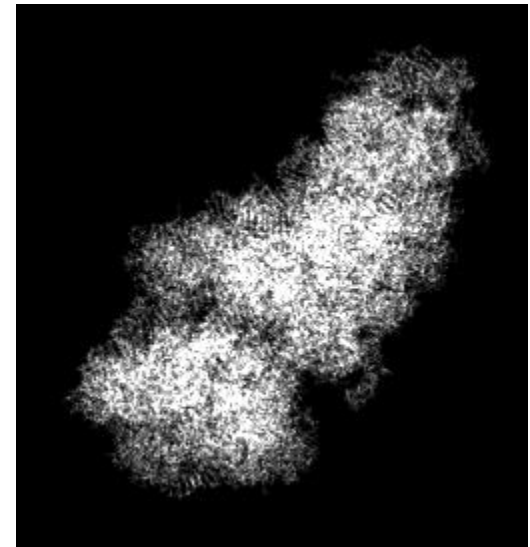
- Projection of a 3D object into 2D
- Each pixel contains the integral of the Coulomb potential along the z-direction



Projection along z



Take the integral of  
each value along  
the z axis



# Contrast Transfer Function

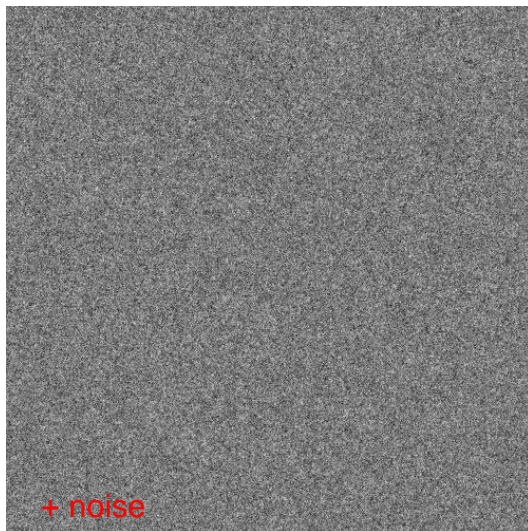
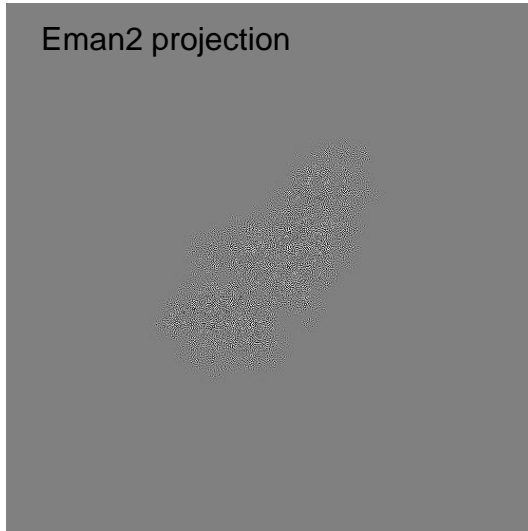
- (in Fourier space)

$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$

$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25C_s\lambda^3 k^4)$$

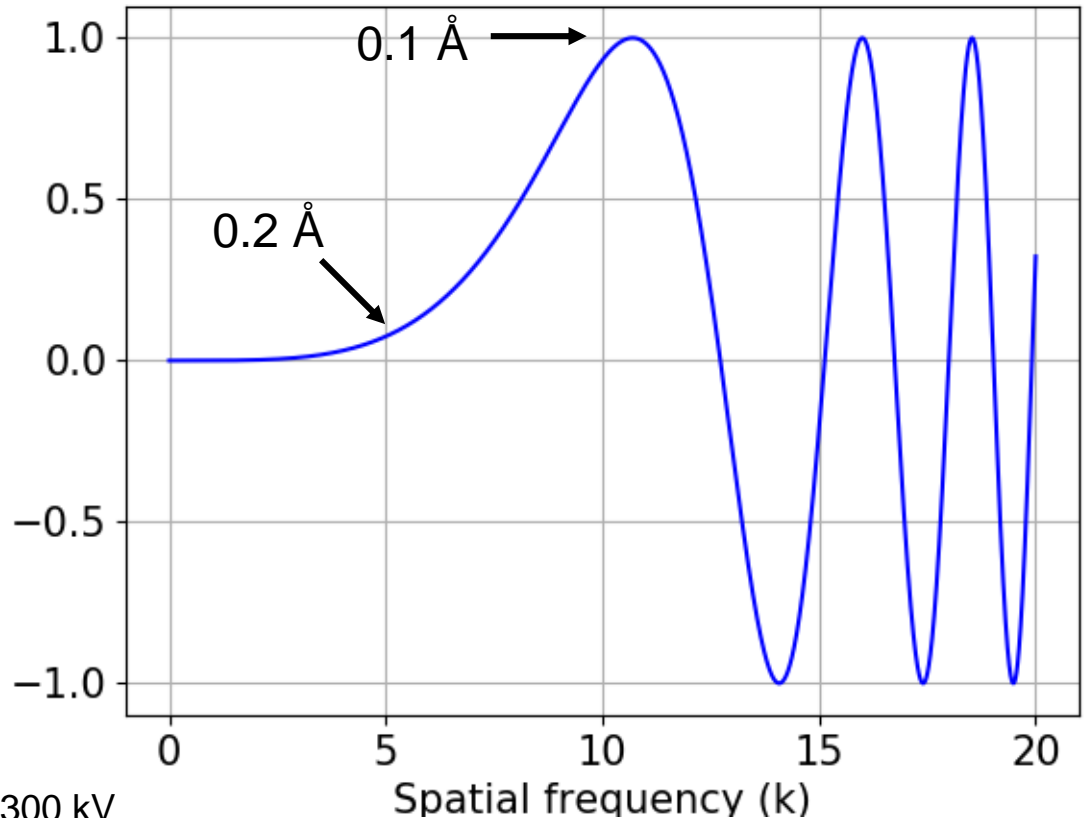
W	amplitude contrast ratio
$\Delta z$	defocus: underfocus is positive
$\lambda$	electron wavelength
$C_s$	spherical aberration
k	spatial frequency

# 300 kV, all zero



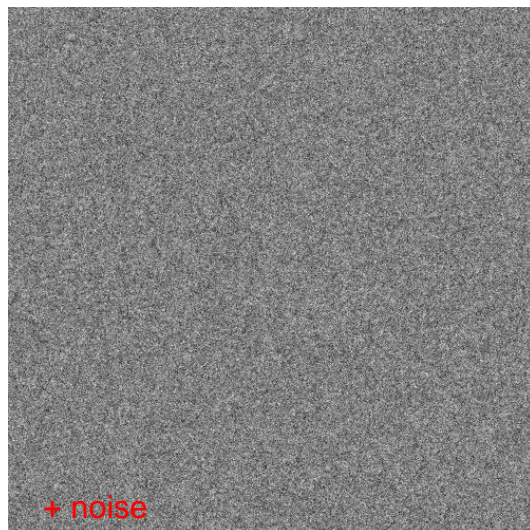
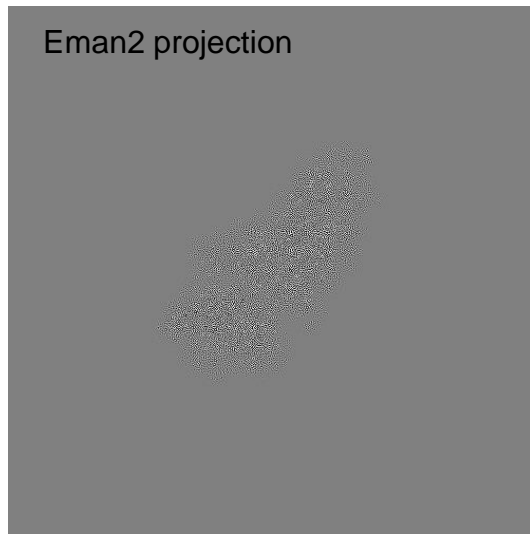
$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$

$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$



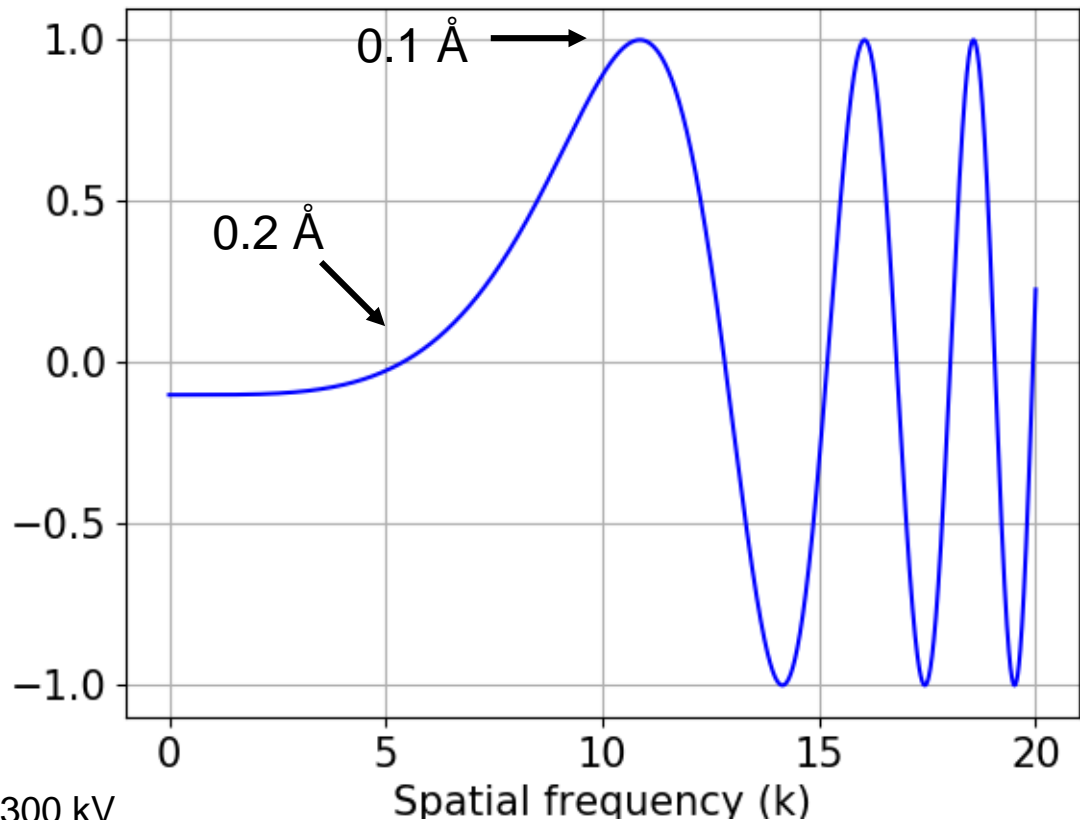
300 kV  
A=0  
B=0 Å<sup>2</sup>  
Cs=0 mm  
Def=0 μm

# 300 kV, 0.1 amp. contrast



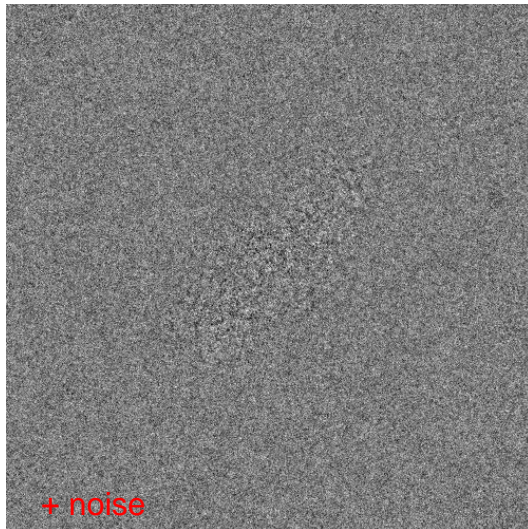
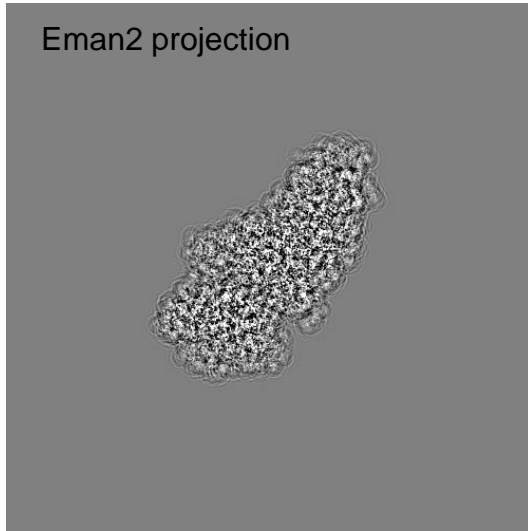
$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$

$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$

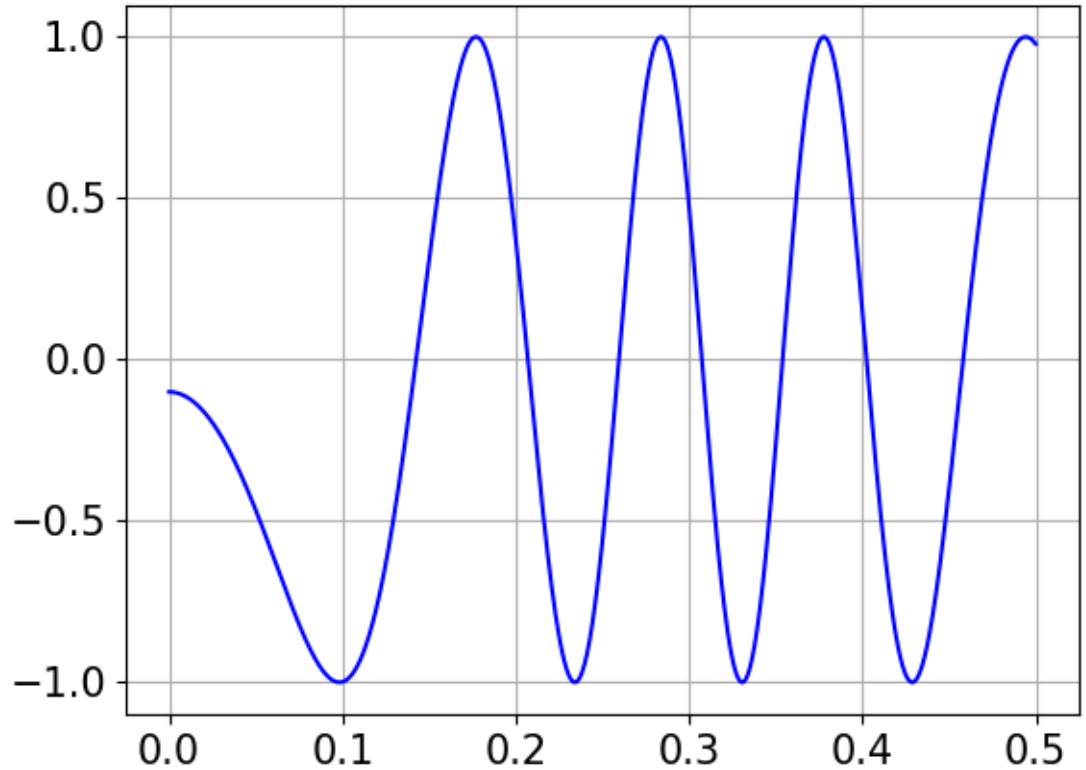


300 kV  
A=0.1  
B=0 Å<sup>2</sup>  
Cs=0 mm  
Def=0 μm

# 0.25 $\mu\text{m}$ defocus



$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$
$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$



300 kV  
A=0.1  
B=0  $\text{\AA}^2$   
Cs=2 mm  
Def=0.25  $\mu\text{m}$

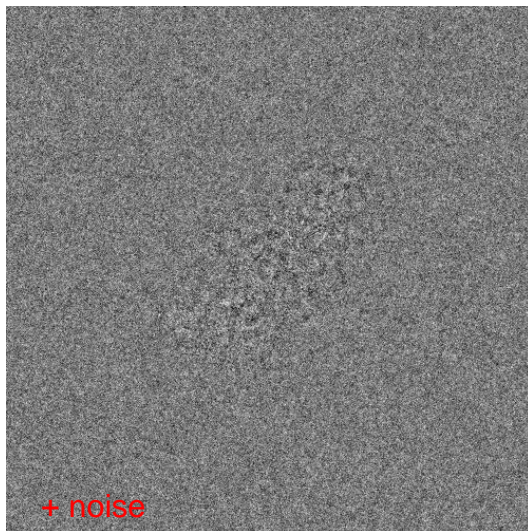
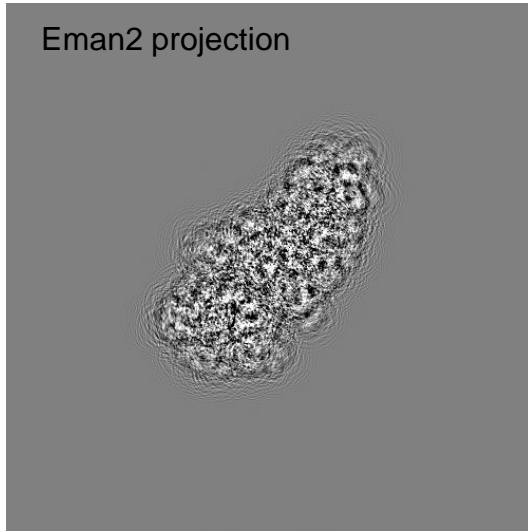
↑  
10  $\text{\AA}$

Spatial frequency (k)

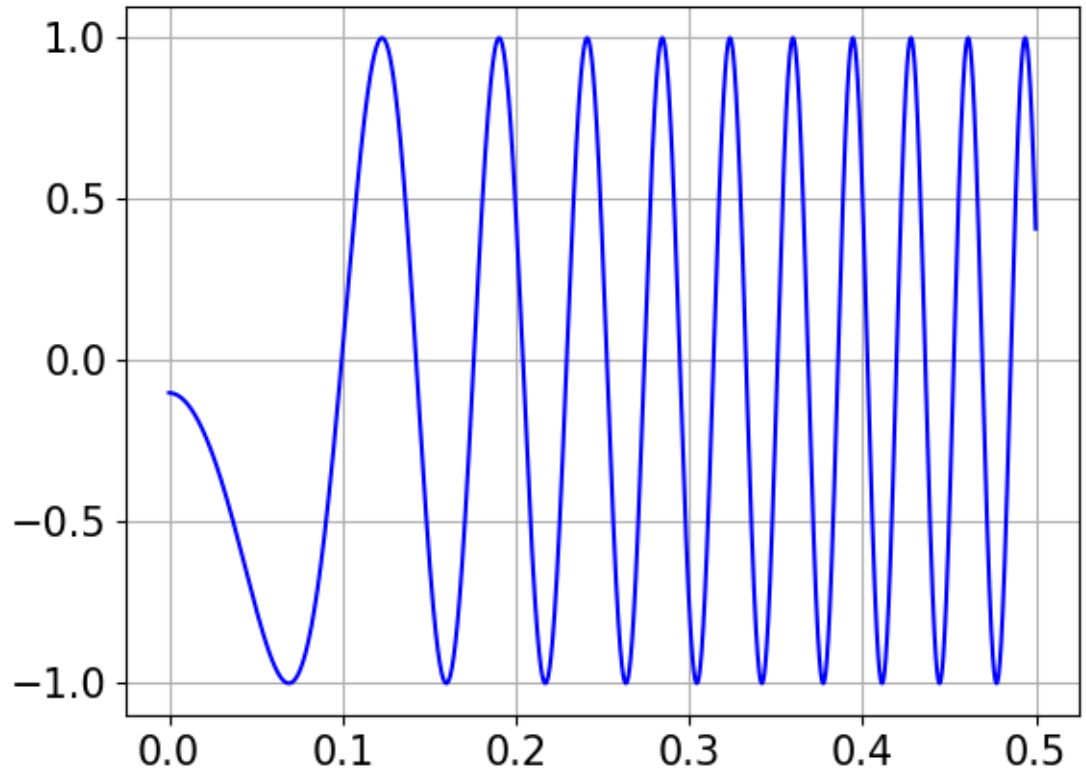
↑  
2  $\text{\AA}$



# 0.5 $\mu\text{m}$ defocus



$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$
$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$

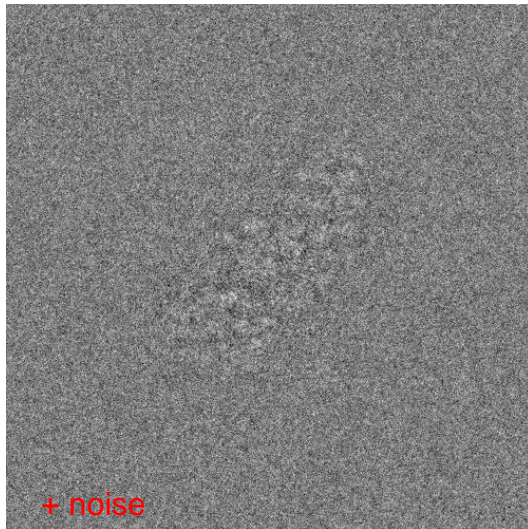
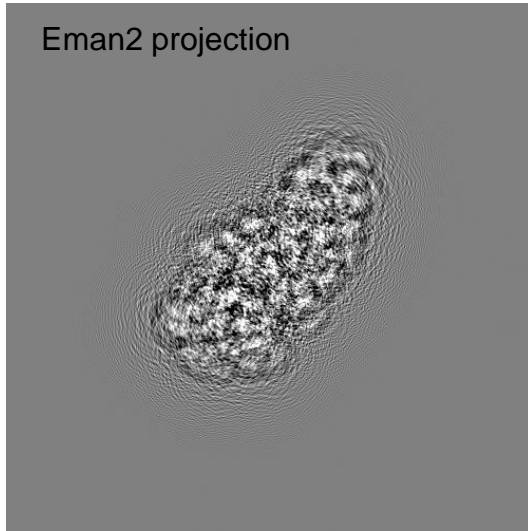


300 kV  
A=0.1  
B=0  $\text{\AA}^2$   
Cs=2 mm  
Def=0.5  $\mu\text{m}$

↑  
10  $\text{\AA}$

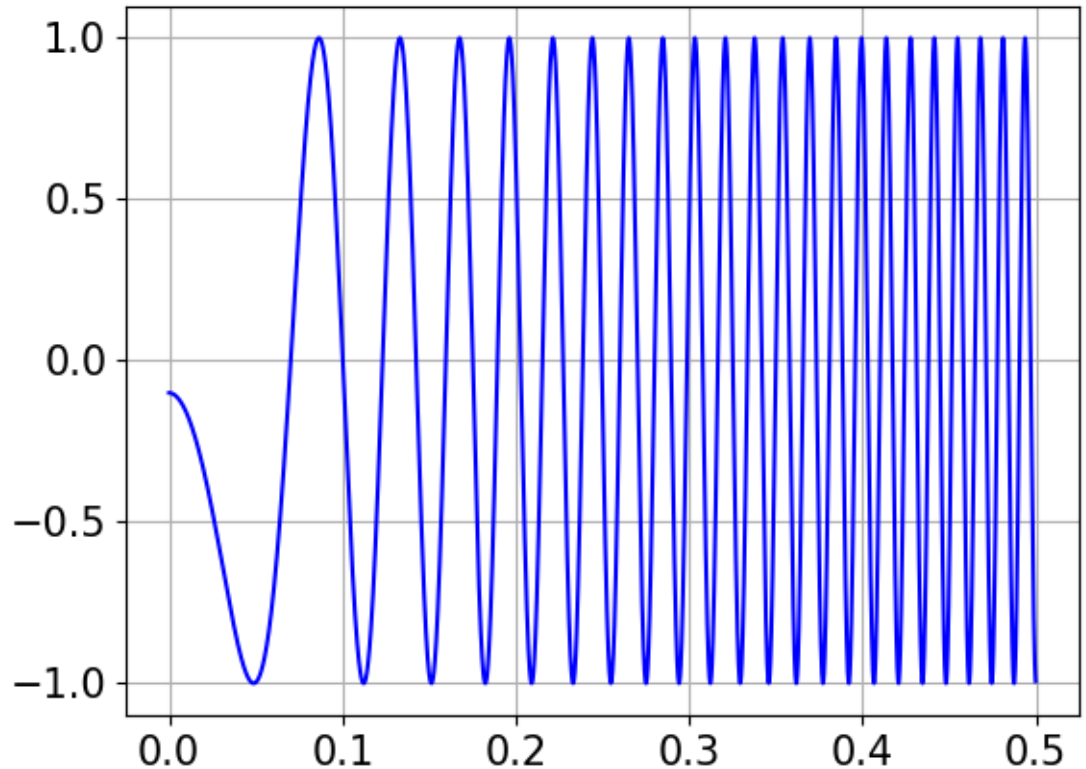
↑  
2  $\text{\AA}$

# 1 $\mu\text{m}$ defocus



$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$

$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$



300 kV  
A=0.1  
B=0  $\text{\AA}^2$   
Cs=2 mm  
Def=1  $\mu\text{m}$

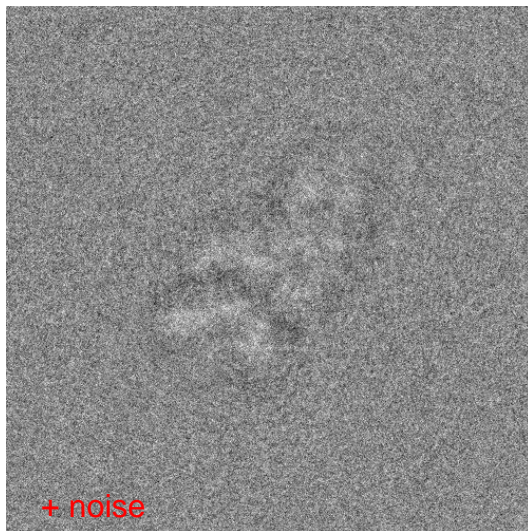
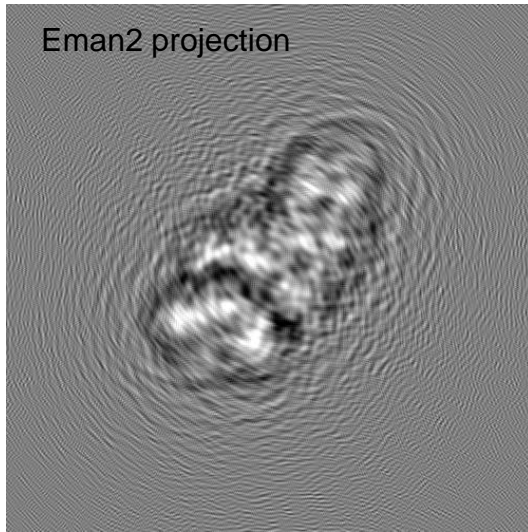
↑  
10  $\text{\AA}$

Spatial frequency (k)

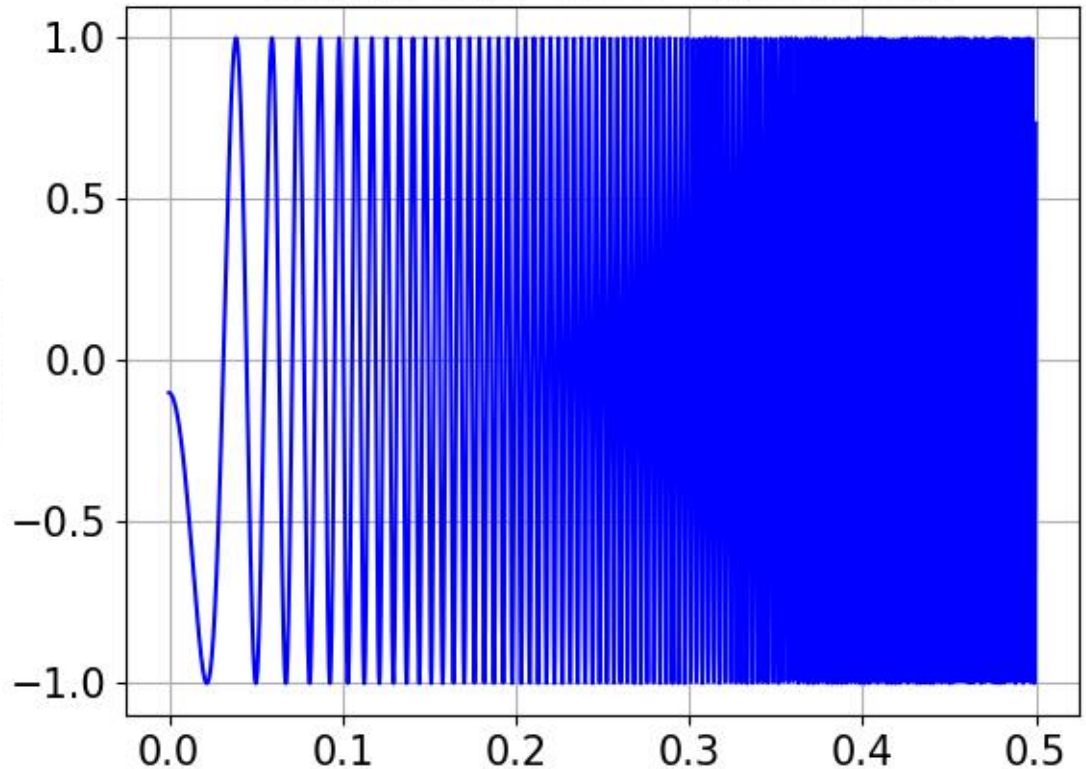
↑  
2  $\text{\AA}$



# 5 $\mu\text{m}$ defocus



$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$
$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$



300 kV  
A=0.1  
B=0  $\text{\AA}^2$   
Cs=2 mm  
Def=5  $\mu\text{m}$

↑  
10  $\text{\AA}$

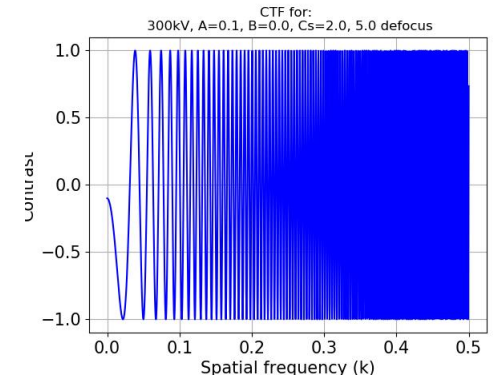
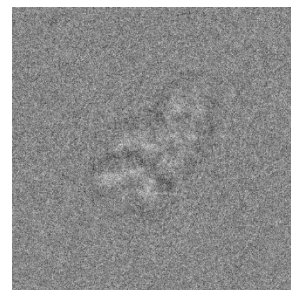
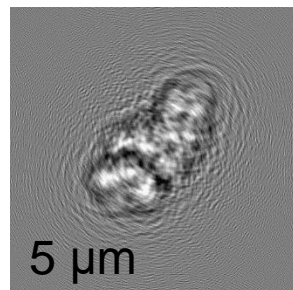
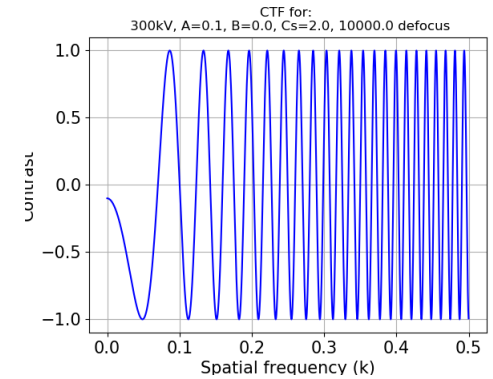
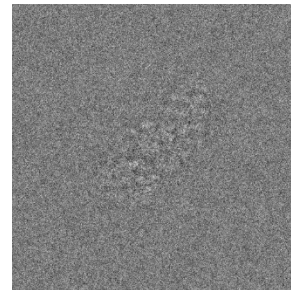
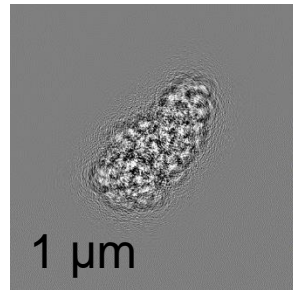
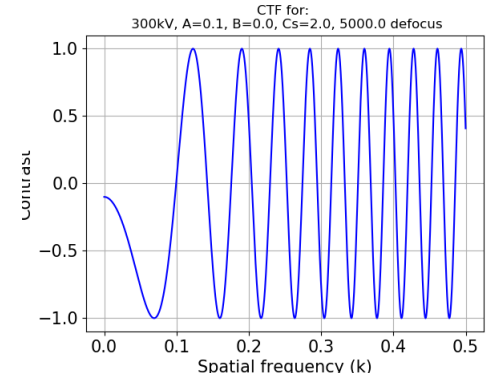
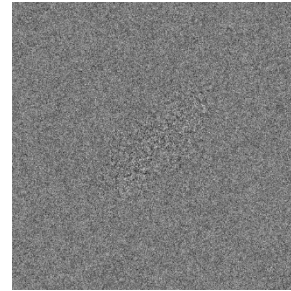
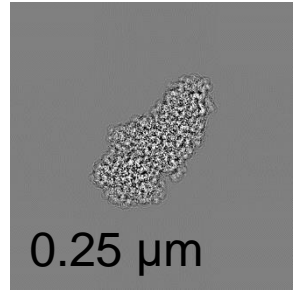
Spatial frequency (k)

↑  
2  $\text{\AA}$

# Change of defocus

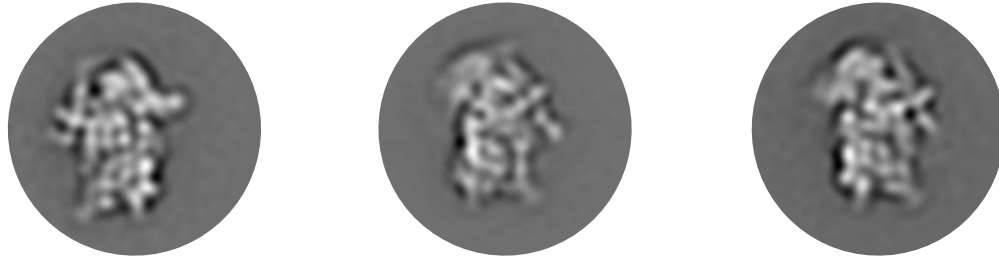
Higher defocus improves contrast

The CTF changes more rapidly

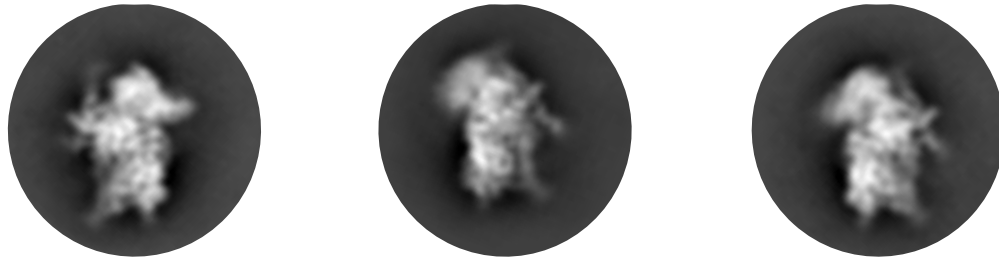


# Why estimating the CTF matters

No correction



CTF corrected

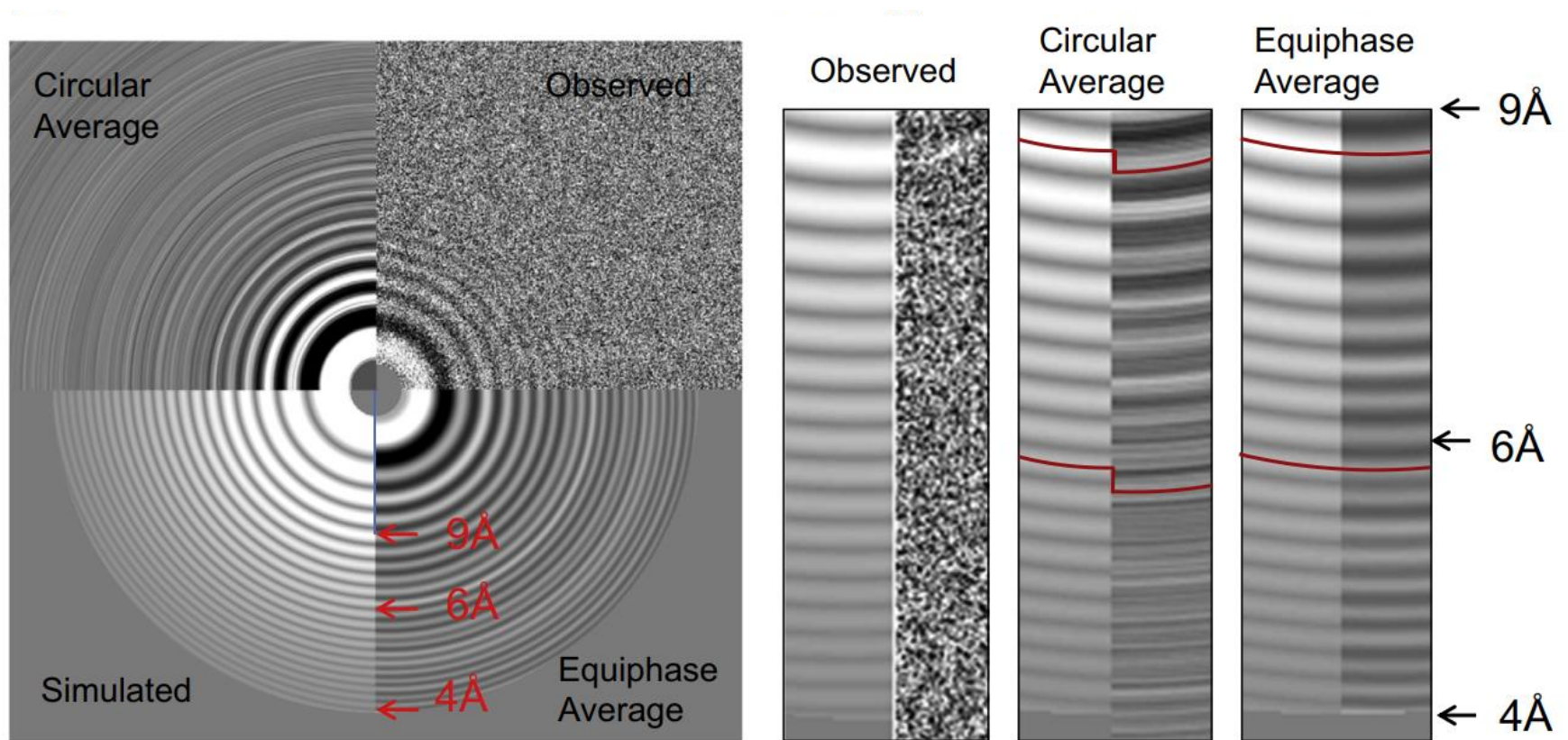


- Various programs estimate the CTF
- CTF estimation is inherently inaccurate



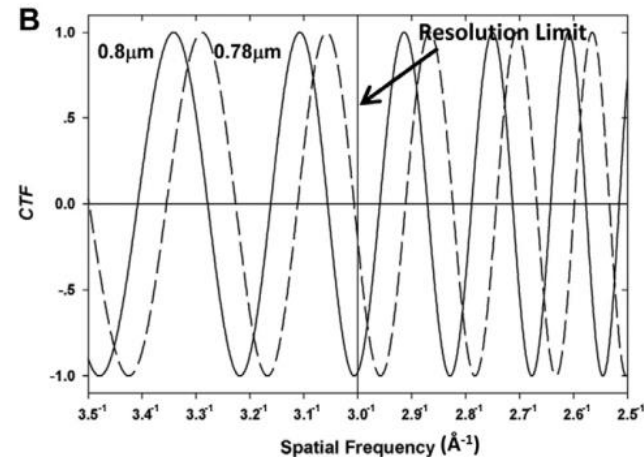
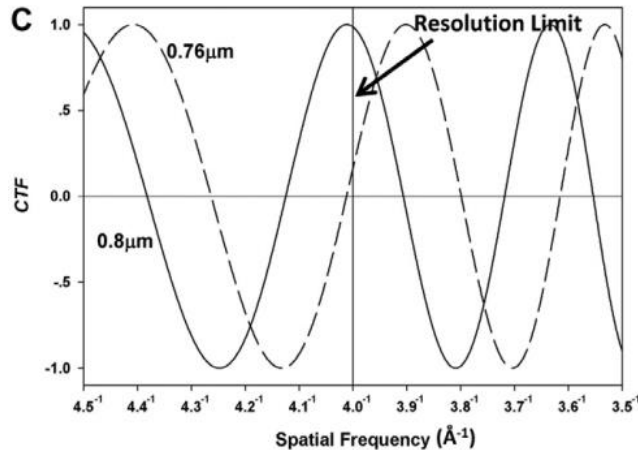
# CTF estimation

- More detail in gCTF, CTFFind4, and Zhu *et al.* 1997



# Effects of estimation error

- Errors of greater than  $\pi/2$  flip phases incorrectly



**Table 3**

Resolution Limit imposed by inaccuracy of defocus determination.

Res. ( $\text{\AA}$ )	100 kV	200 kV	300 kV	400 kV
2.0	54 $\text{\AA}$	80 $\text{\AA}$	102 $\text{\AA}$	122 $\text{\AA}$
3.0	122 $\text{\AA}$	179 $\text{\AA}$	228 $\text{\AA}$	274 $\text{\AA}$
4.0	216 $\text{\AA}$	319 $\text{\AA}$	406 $\text{\AA}$	488 $\text{\AA}$
7.0	662 $\text{\AA}$	976 $\text{\AA}$	1244 $\text{\AA}$	1494 $\text{\AA}$

# Goals for today

- Image formation in EM
- The contrast transfer theory
- CTF equation
- Effect of various parameters on the CTF
- Why CTF estimation matters
- Envelope functions

# Envelope functions

Finite source size  
(source:  $q$ )

$$E_{pc}(k) = \exp [-\pi^2 q^2 (k^3 C_s \lambda^3 - \Delta z k \lambda)^2],$$

Energy spread  
( $\delta z$  defocus variation)

$$E_{es}(k) = \exp \left[ -\frac{1}{16 \ln 2} \pi^2 \delta z^2 k^4 \lambda^2 \right],$$

MTF of film

$$E_f(k) = 1/[1 + (k/k_f)^2],$$

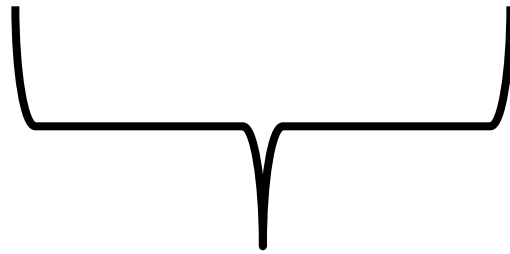
Generic envelope func.  
(drift, specimen charging,  
multiple scattering)

$$E_g(k) = \exp [-(k/k_g)^2],$$

$$I(\mathbf{k}) = E_{pc}(k) E_{es}(k) E_f(k) E_g(k) H(k) \Phi(\mathbf{k}) + N(\mathbf{k}).$$

# B-factor: the generic envelope function

$$I(\mathbf{k}) = E_{pc}(k)E_{es}(k)E_f(k)E_g(k)H(k)\Phi(\mathbf{k}) + N(\mathbf{k}).$$



$$e^{-Bk^2}$$

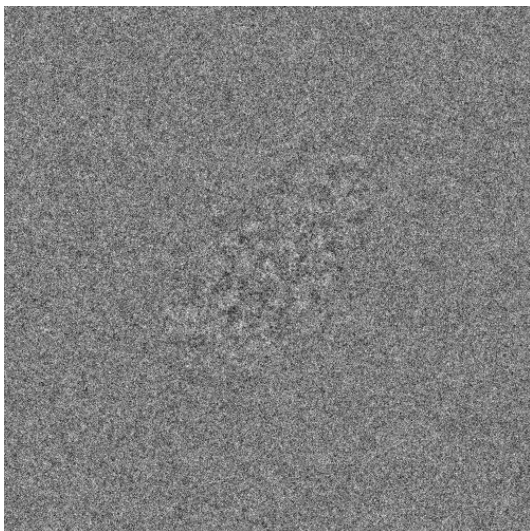
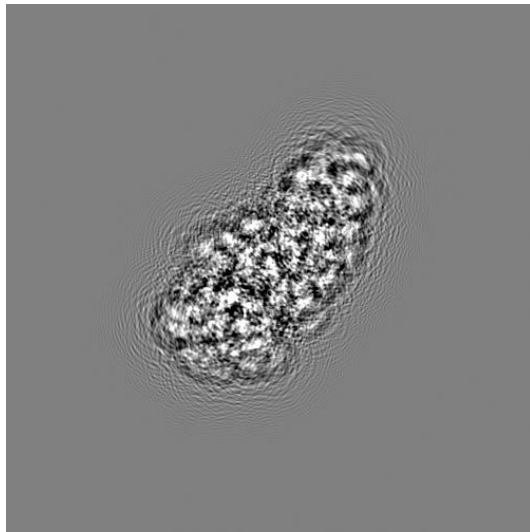
“[this] formulation is in conflict with the theory of partial coherence, according to which the damping term due to finite source size is defocus dependent” (Frank, 2006)

Finite source size  
(source: q)

$$E_{pc}(k) = \exp [-\pi^2 q^2 (k^3 C_s \lambda^3 - \Delta z k \lambda)^2],$$

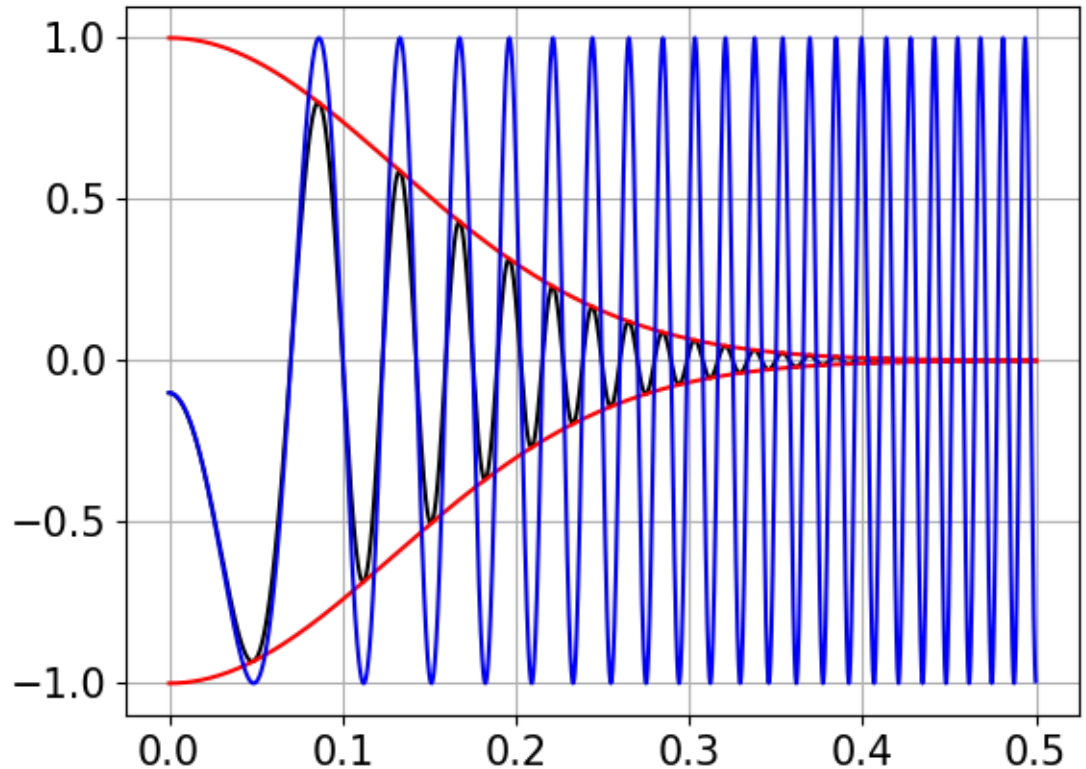


# B-factor: 30 Å<sup>2</sup>



$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$

$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$

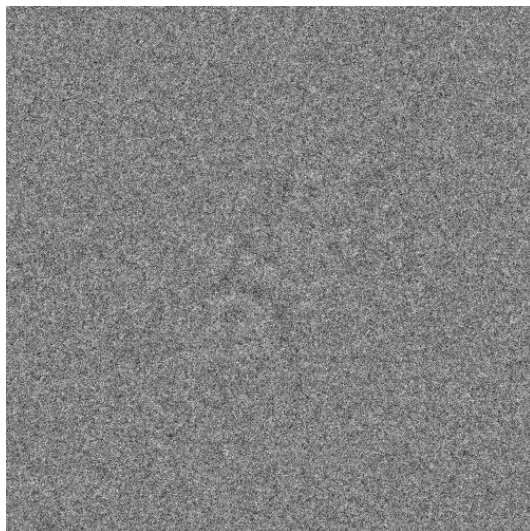


300 kV  
A=0.1  
B=30 Å<sup>2</sup>  
Cs=2 mm  
Def=0.25 μm

↑  
10 Å

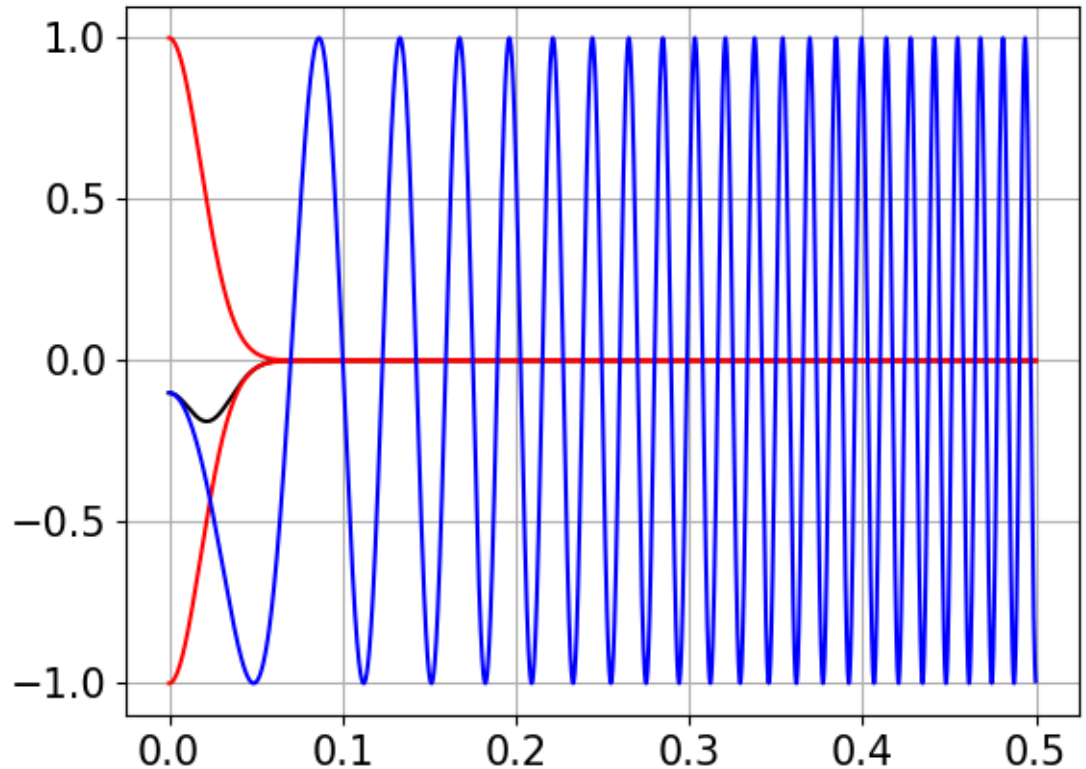
↑  
2 Å

# B-factor: 1500 Å<sup>2</sup>



$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$

$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$



300 kV  
A=0.1  
B=1500 Å<sup>2</sup>  
Cs=2 mm  
Def=0.25 μm

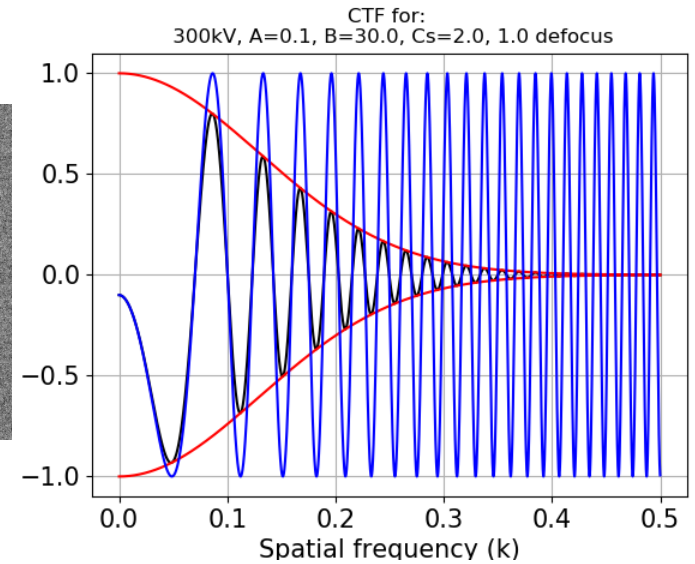
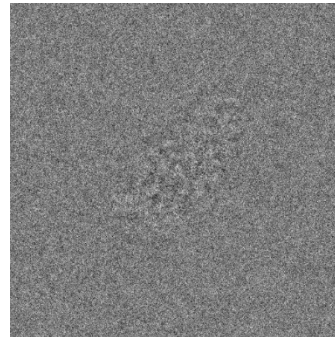
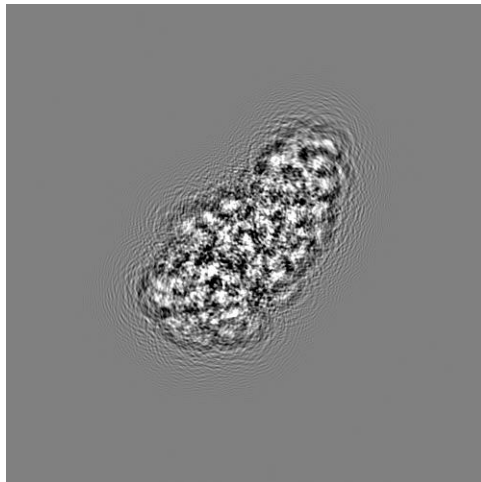
↑  
10 Å

Spatial frequency (k)

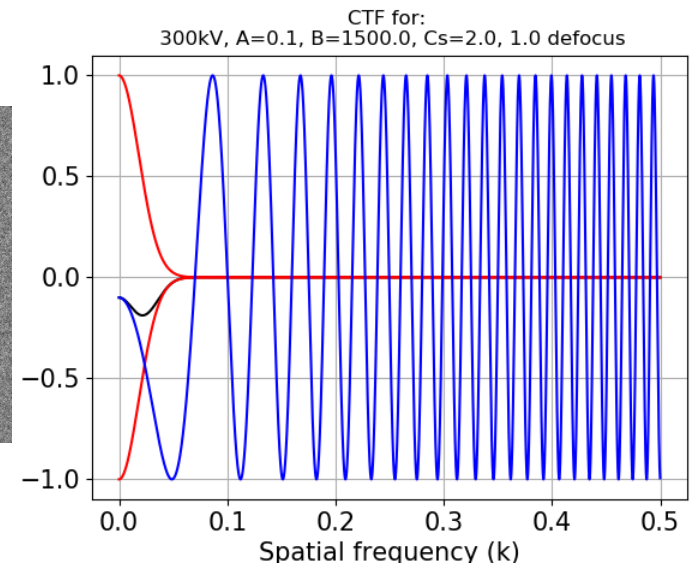
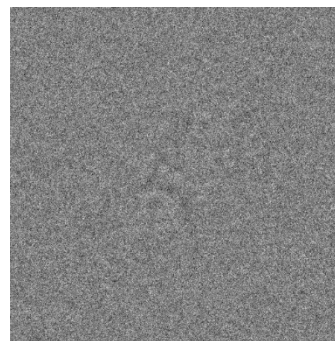
↑  
2 Å

# B-factor

30 Å<sup>2</sup>



1500 Å<sup>2</sup>

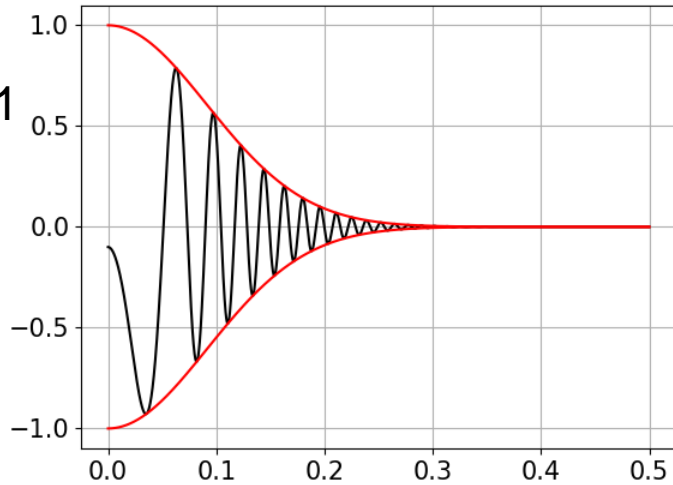


# Effect of different acceleration voltage

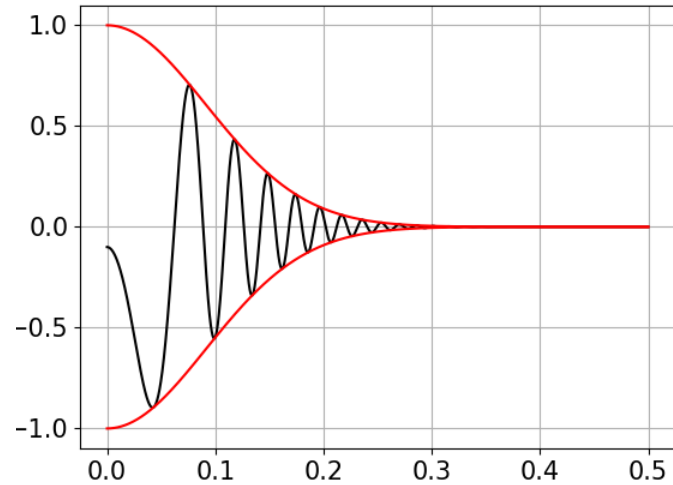
$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$

$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$

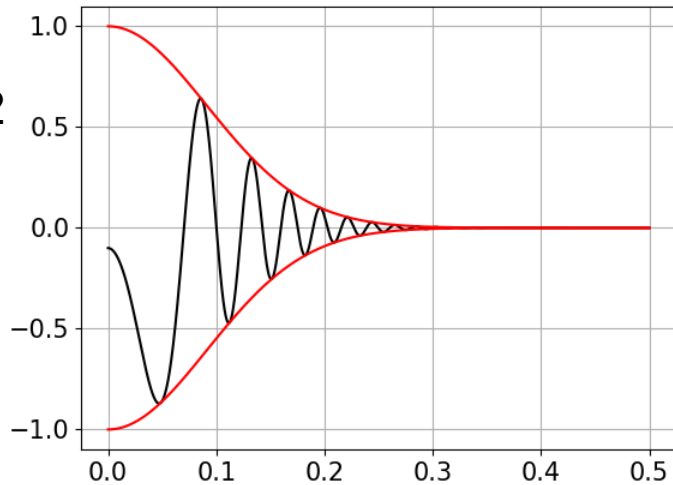
100 kV  
Area= 681



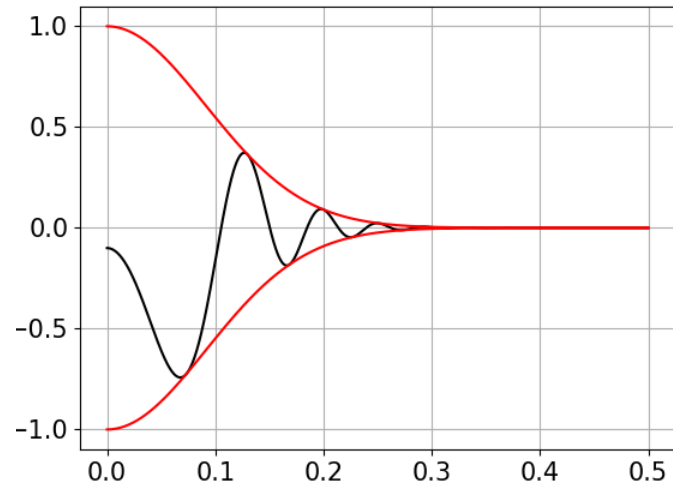
200 kV  
Area=670



300 kV  
Area=662



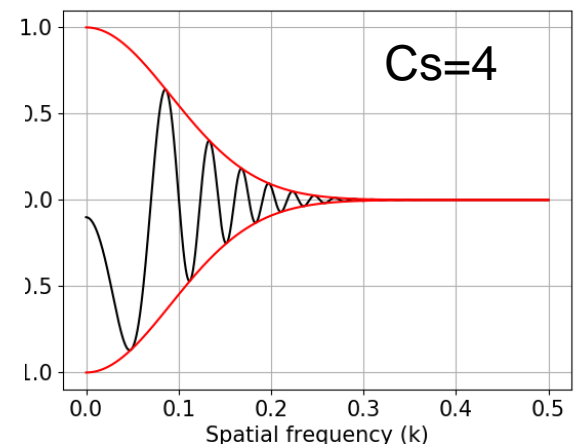
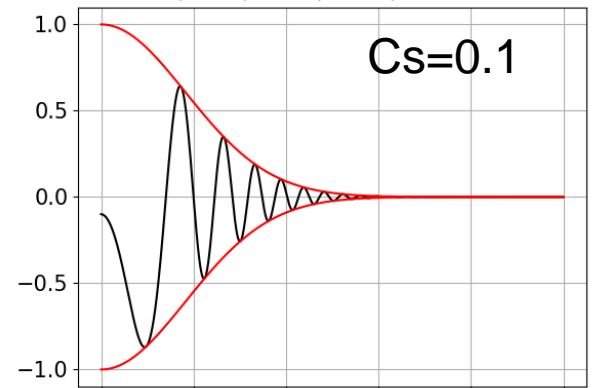
1000 kV  
Area=650



# Conclusion

$$H(k) = 1[\sin\gamma(k) - W\cos\gamma(k)]$$
$$\gamma(k) = 2\pi(-0.5\Delta z\lambda k^2 + 0.25Cs\lambda^3 k^4)$$

- Everything is a trade off
- Defocus increases contrast but makes CTF estimation more difficult (error prone)[*citation needed!*]
- $\Delta z$ ,  $\lambda$ ,  $Cs$  change the CTF
  - NB: Why  $Cs$  doesn't matter much...



# How to investigate further

- Eman2

e2pdb2mrc.py

PDB → MRC

e2project3d.py

Make projections

e2filtertool.py

math.simultatectf

e2proc2d.py

apply in bulk

- Script got CTF examples

ctf\_simulation\_v0.4.py

[github.com/zubengithub/CTF](https://github.com/zubengithub/CTF)



# Next Three sessions

- Grids
- Electron-specimen interactions
- Data processing strategies (MC2, gCTF, CTFFind4, particle polishing)